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THE "RATCHET PRINCIPLE"

AND PERFORMANCE INCENTIVES

by

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SUMMARY

The use of current performance as a partial basis for setting future targets is an almost universal feature of economic planning. This "ratchet principle", as it is sometimes called, creates a dynamic incentive problem for the enterprise. Higher rewards from better current performance must be traded off against the future assignment of more ambitious targets. The problem of the enterprise is formulated in this paper as a multi-period optimization model incorporating an explicit feedback mechanism for target setting. Fortunately, it is very easy to characterize an optimal solution. The incentive effects of the ratchet principle can be fully analyzed in simple economic terms.

INTRODUCTION

Understanding how incentive systems work is an important task of economic theory. To date, most analyses of reward structures have been essentially static.[1] For some situations this is not a serious limitation. Unfortunately, certain important incentive issues have an inherently dynamic character that cannot even be formulated, let alone analyzed, in a timeless framework.[2]

Consider the "standard reward system". Let y be a performance indicator for the enterprise. Usually y will symbolize output, but profits, cost, or productivity might be appropriate in some contexts.

Let the target, goal, or quota be denoted q. In a standard reward system, the variable component of an enterprise's bonus is typically proportional to the difference between y and q, or at least such a formulation is a decent approximation for most analytical purposes.

There are two basic incentive problems associated with a standard reward system. The immediate difficulty is essentially a static problem of misrepresentation. It is in the interest of the manager (or worker) to convince his superiors that y is likely to be small, thereby entitling him to a lower q and a bonus which is easier to attain.

The dynamic incentive problem, on which this paper concentrates, arises out of the well-known tendency of planners to use current performance as a criterion in determining future goals. This tendency has sometimes been called the "ratchet principle" of economic planning because current performance acts like a notched wheel in fixing the point of departure for next period's target.[3]

Operation of the ratchet principle is widespread in planning or regulatory contexts ranging from the determination of piecework standards for individual workers to fixing budgets or output quotas for large bureaucracies. In such situations the agent faces a dynamic trade-off between present rewards from better current performance and future losses from the assignment of higher targets.

The ratchet principle necessitates a multiperiod statement of the enterprise's problem, which at first glance appears to be very messy. One of the principal aims of this paper is to show that in fact, under a not unreasonable formulation, the enterprise's dynamic problem can be

easily solved and given a neat economic interpretation. The effect of the ratchet principle on economic performance, and how that effect depends on various factors, is simple to state and analyze.

THE MODEL

The economic unit whose behavior we will be studying is called an "enterprise". This term is employed in a broad sense because, depending on the context, it might pertain to an individual worker, an intermediate sized department, or a giant sector. The enterprise operates in a planned environment where it and the planners mutually interact. Such an environment might be found within multidivisional private firms, government or quasi-public organizations, or nationalized branches of the economy.

Let $t = 1, 2, 3, \ldots$ index the plan period. During any period, enterprise performance will typically be affected by the plan target for that period and will in turn influence the formation of next period's target.

The planning period discount rate is denoted r. That is, next period's gains are transformed into this period's by the factor $\frac{1}{1+r}$. If ρ is the instantaneous force of interest and ℓ is the length of the plan period,

$$\frac{1}{1+r} = e^{-\rho \ell} ,$$

or,

$$r = e^{\rho \ell} - 1. \tag{1}$$

Thus, r might be larger or smaller depending on the length of the review lag ℓ and the interest rate ρ.

The variable y_t will symbolize performance of the enterprise in period t. It is perhaps easiest to think of inputs being exogenously determined and let y_t denote output (for convenience, this will be our primary interpretation), but profits or productivity could also be accomodated. In some contexts it may be more appropriate to envision a fixed task given in period t and have y_t, here a negative output, stand for minus the cost of accomplishing the task.

When the enterprise performs at level y_t in period t, it incurs net disutility, loss, or cost $C_t(y_t)$ exclusive of any bonus payments received. The cost of performance is typically time dependent because the means available for meeting plan assignments.treated here as exogenously predetermined, may differ from period to period. A growing enterprise will frequently have an ever increasing capacity to work with.

It is postulated that

$$C_t'' \geq 0, \tag{2}$$

which ensures that second order conditions are always met.

The performance target in period t is denoted q_t. We assume that the bonus received by the enterprise can be written in the form

$$b(y_t - q_t)$$

where b is a bonus coefficient.

If q_t were exogenously fixed for all t, the enterprise in period t

would seek to maximize over y_t the total gain[4]

$$b(y_t - q_t) - C_t(y_t),$$

resulting in performance level $\tilde{y}_t$ satisfying

$$C_t'(\tilde{y}_t) = b.$$

A more realistic scenario would have q_t determined by some version of the ratchet principle. The specific form postulated here is:

$$q_t - q_{t-1} = \delta_t + \lambda(y_{t-1} - q_{t-1}) \quad (3)$$

The independent increment δ_t represents how much the target would be changed in period t if last period's target were exactly met. For every notch that last period's performance exceeded last period's target, this period's target will be pushed up by an additional λ notches. The adjustment coefficient λ is treated as a behavioral parameter of the planners which quantifies the strength of the ratchet principle.

An instructive way of rewriting (3) is

$$q_t = \lambda y_{t-1} + (1-\lambda)q_{t-1} + \delta_t.$$

This period's target is a weighted average of last period's performance and last period's target, plus an independent increment. The weight on last period's performance is the adjustment coefficient λ.

In the target setting process, δ_t will be treated as a random variable independently distributed from one period to the other. Actually it is possible to incorporate into the model more general forms of uncertainty without altering the main results, but the notation would become

too unwieldy.

With $\{\delta_t\}$ independently distributed, under the given target setting procedure all relevant statistical history at time t is summarized by the state variable q_t. A <u>decision rule</u>

$$y_t(q_t)$$

expresses the performance level at time t as a function of the assigned target. The set of decision rules $\{y_t(q_t)\}$ results in an expected value to the enterprise of

$$V(\{y_t\}) = \sum_{t=1}^{\infty} [b(y_t(q_t) - q_t) - C_t(y_t(q_t))] \left(\frac{1}{1+r}\right)^t , \tag{4}$$

where

$$q_t = (1-\lambda)q_{t-1} + \lambda y_{t-1} + \delta_t, \tag{5}$$

$$q_o = \bar{q}_o,\ y_o = \bar{y}_o \text{ (initial conditions).} \tag{6}$$

The expectation operator E is taken over the random variables $\{\delta_t\}$.

Given the passive target setting behavior of the planners, the problem of the enterprise is to maximize expected present discounted value, or to find a set of <u>optimal</u> decision rules $\{y_t^*(q_t)\}$ satisfying

$$V(\{y_t^*(q_t)\}) = \max_{\{y_t(q_t)\}} V(\{y_t(q_t)\}). \tag{7}$$

This problem is representative of a class of models which attempt to characterize optimal behavior in the presence of a regulatory lag. We assume that the problem (4)-(7) is well defined and that an optimal solution exists.

The issue of existence is not of interest in its own right, and anyway it is not difficult to specify sufficient conditions for (4)-(7) to be a meaningful problem.

THE RATCHET EFFECT

At first glance it might appear that problem (4)-(7) is difficult to solve. In fact, an exceedingly straightforward solution concept is available.

The following theorem is the basic result of the present paper:

<u>y_t^* is the optimal performance level in period t if and only if it satisfies</u>

$$\underline{C_t'(y_t^*) = \frac{b}{1 + \frac{\lambda}{r}}} \tag{8}$$

Note the extreme simplicity of an optimal policy. Rule (8) is completely myopic. y_t^* depends only on the parameters b and λ/r, and on the current cost function.

If costs are time invariant so that

$$C_t(y_t) = C(y_t),$$

the optimal strategy is to always perform at the same constant level y* satisfying

$$C'(y^*) = \frac{b}{1 + \frac{\lambda}{r}} .$$

Perhaps it is easiest to think of $\{y_t^*\}$ as those performance levels which would be elicited if the same hypothetical "ratchet price"

$$p = \frac{b}{1 + \frac{\lambda}{r}} \tag{9}$$

were offered for each period's output. Should it seek to

$$\underset{y_t}{\text{maximize}} \quad py_t - C_t(y_t),$$

the enterprise, by setting the marginal cost of output equal to the ratchet price, would automatically attain the optimal solution y_t^*. The entire effect of the ratchet principle can be thought of as transmitted through the ratchet price. The higher the ratchet price, the higher the optimal output in each period.

Note that with $\lambda > 0$, the ratchet price p is lower than the bonus coefficient b. The ratchet effect diminishes performance in each period.

Comparative statics are easily performed; p, and hence y_t^*, is lower as b is lower, as λ is higher, or as r is lower. The ratchet effect varies directly with the adjustment coefficient, as of course it should. There is also a stronger ratchet effect as r is smaller. From formula (1), shorter review lags or lower interest rates will cause the enterprise to weigh more strongly the adverse effects of over-zealous present performance on raising future targets.

It is instructive to look at extreme values of λ and r. There is no ratchet effect, $p \to b$, as either $\lambda \to 0$ or $r \to \infty$. There is a maximal ratchet effect equivalent to a zero price of output, $p \to 0$, as either $\lambda \to \infty$ or $r \to 0$. Such extreme results accord well with economic intuition.

Taking the ratchet principle as given, the aim of this paper has been to investigate effects on enterprise performance. Naturally the model is a gross oversimplification of reality. Even so, it seems to capture the main ingredients of the dynamic incentive problem, and it does allow a sharp quantification of the basic tradeoffs involved in the ratchet effect. The possibility of explicitly constructing an optimal solution makes the problem analyzed here a natural preliminary to more general formulations. And the present model may even be a reasonable description of some planning or regulatory situations.

PROOF OF THE OPTIMAL POLICY

Consider any decision rule $\{y_t(q_t)\}$. For notational convenience we will henceforth drop the explicit dependence on q_t and simply write y_t to stand for $y_t(q_t)$.

It is not difficult to verify that the solution of (5), (6) is

$$q_s = q_o(1-\lambda)^s + \sum_{t=0}^{s-1} (\lambda y_t + \delta_{t+1})\ (1-\lambda)^{s-1-t}. \tag{10}$$

Using (10), expression (4) (the expected value of the decision rule) becomes

$$V = E \sum_{s=1}^{\infty} [b(y_s - q_o(1-\lambda)^s - \sum_{t=0}^{s-1}(\lambda y_t + \delta_{t+1})(1-\lambda)^{s-1-t})$$

$$- C_s(y_s)]\ (\tfrac{1}{1+r})^s \tag{11}$$

Changing the order of summation, (11) can be rewritten as

$$V = E \sum_{t=1}^{\infty} [b(y_t - q_o(1-\lambda)^t - C_t(y_t)] \left(\frac{1}{1+r}\right)^t$$

$$- E \sum_{t=0}^{\infty} b(\lambda y_t + \delta_{t+1}) \sum_{s=t+1}^{\infty} (1-\lambda)^{s-1-t} \left(\frac{1}{1+r}\right)^s. \tag{12}$$

Eliminating the expectation sign where it is superfluous and using the fact that

$$\sum_{s=t+1}^{\infty} (1-\lambda)^{s-1-t} \left(\frac{1}{1+r}\right)^s = \frac{1}{\lambda+r}\left(\frac{1}{1+r}\right)^t,$$

rewrite (12) as

$$V = \sum_{t=1}^{\infty} \left[\frac{b}{1 + \frac{\lambda}{r}} y_t - C_t(y_t)\right]\left(\frac{1}{1+r}\right)^t - K, \tag{13}$$

where

$$K \equiv by_o\left(\frac{\lambda}{\lambda+r}\right) + \sum_{t=1}^{\infty} bq_o\left(\frac{1-\lambda}{1+r}\right)^t$$

$$+ E \sum_{t=0}^{\infty} b\delta_{t+1}\left(\frac{1}{\lambda+r}\right)\left(\frac{1}{1+r}\right)^t. \tag{14}$$

From (14), K is a constant independent of $\{y_t\}$. The variable part of (13) is additively separable in functions of y_t. Hence, (13) will be maximized if and only if in each period t, y_t is selected to maximize

$$\frac{b}{1 + \frac{\lambda}{r}} y_t - C_t(y_t).$$

With the second order condition (2) and no constraint on the domain of y_t, the optimal value y_t^* must satisfy (8). This concludes our proof of the form of an optimal policy.

FOOTNOTES

[1] See, for example, Weitzman [1976] and the references cited there.

[2] This has been recognized in the earlier work of Yunker [1973], Weitzman [1976], and Snowberger [1977].

[3] The term "ratchet principle" was coined by Berliner. For descriptions see Berliner [1957] pp. 78-80, Bergson [1964] pp.75-76. Zielinski [1973] p. 122, Berliner [1976] pp. 408-409.

[4] We are implicitely assuming that the one period gain can be written as bonus income minus a disutility of effort term which is independent of income.

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